Math 72 5.6

Objectives

1) Factor completely using appropriate methods.
   - ☆ review methods
   - ☆ recognize when to do each method
   - ☆ recognize when answer is completely factored.

2) ☆ factor by substitution.

IMPORTANT

When factoring an <u>expression</u> (no = sign), the final result must equal the original question.

We cannot multiply or divide by any number except 1.

Factors which have been factored out are part of the final answer.

Process for Factoring over the Integers

Main concept: Factoring "un-does" multiplying. Multiplying "un-does" factoring.

When in doubt, multiply your result. You should get your original expression (or a simplified version of it).

When factored, all addition and subtraction must be inside parentheses.

Step 0: Get organized.

Arrange the terms in standard form, descending from the leading (highest-degree) term first.

(If there is more than one variable, choose a variable and arrange in descending order by that variable.)

Step 1: GCF. Factor out the greatest common factor from all terms.

Step 2: Count the terms.

Terms are separated by add or subtract symbols which are outside parentheses.

Step 3: 2-term patterns.

3a: Sum of squares: $a^2 + b^2$ is prime.

3b: Difference of squares: $a^2 - b^2 = (a-b)(a+b)$

3c: Sum of cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

3d: Difference of cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

[For 3c & 3d, the acronym S.O.A.P. can be used for the three signs: **S**ame – **O**pposite – **A**lways **P**ositive]

Step 4: 3-term patterns:

4a: Perfect Square Trinomial (sum): $a^2 + 2ab + b^2 = (a+b)^2$

4b: Perfect Square Trinomial (difference): $a^2 - 2ab + b^2 = (a-b)^2$

4c: Leading coefficient $a = 1$: $x^2 + bx + c$;

- Find two numbers that multiply to *c* and add to *b* using guess-and-check or magic X.

4d: Leading coefficient $a \neq 1$: $ax^2 + bx + c$;

- Use guess-and-check by finding numbers for the first terms that multiply to *a* and numbers for the second terms that multiply to *c*, or
- Use the "double magic X" by finding two numbers that multiply to the product *ac* and add to *b*, then use these to rewrite the middle term and factor by grouping.

4e: If the expression is quadratic in form, $a(garbage)^2 + b(garbage) + c$,

- Substitute *u=garbage* to get a true quadratic, factor using *u* and one of the methods 4a-4d, then replace *u* by *garbage*, simplify inside parentheses. Check for greatest common factor.

Step 5: 4-terms, factor by grouping.

5a: Two groups of two terms: GCF first group, GCF second group, binomial GCF.

5b: Three terms make a perfect square trinomial minus a perfect square, then factor difference of squares.

Step 6: Factor each factor. Continue factoring until every factor is prime.

Examples of prime factors:

Monomial GCF: $-3xy^3$ or $2x$
Linear Binomial: $(x-4)$ or $(2x-3)$
Sum of squares: (x^2+4)
Sum of a square with a non-square: (x^2+7)
Difference of a square with a non-square: (x^2-8)
A sum or difference with one square and one cube: (x^2-y^3)
The trinomial factor from a sum or difference of cubes: (x^2-2x+4) or (x^2+2x+4)

Examples of factors that are not prime and can be factored:

Linear binomial with a GCF: $(2x-4)=2(x-2)$
Quadratic binomial with GCF: $(14x^2+7)=7(2x^2+1)$
A perfect square trinomial: $(x^2-2xy+y^2)=(x-y)(x-y)=(x-y)^2$
A sum of cubes: $(x^3+y^3)=(x+y)(x^2-xy+y^2)$

Difference of fourth powers (difference of squares twice, with a prime sum of squares):
$(x^4-y^4)=(x^2-y^2)(x^2+y^2)=(x-y)(x+y)(x^2+y^2)$

Difference of sixth powers (difference of squares, then difference of cubes and sum of cubes):
$(x^6-y^6)=(x^3-y^3)(x^3+y^3)=(x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)$

The number 1 is a perfect square, a perfect cube, a perfect 4th power, … any power of 1 is 1.
$(x^4-1)=(x^2-1)(x^2+1)=(x-1)(x+1)(x^2+1)$
$(x^3+1)=)(x+1)(x^2-x+1)$

The number 64 is both a perfect square ($64=8^2$) and a perfect cube ($64=4^3$):
$(x^3+64)=)(x+4)(x^2-4x+16)$
$(x^4-64)=(x^2-8)(x^2+8)$

The number 16 is both a perfect square ($16=4^2$) and a perfect 4th power ($16=2^4$):
$(x^4-16)=(x^2-4)(x^2+4)=(x-2)(x+2)(x^2+4)$

Common Mistake:
A perfect cube is not the same as a sum of cubes or a difference of cubes:
$x^3-6x^2+12x-8=(x-2)^3$ because when multiplying $(x-2)^3=(x-2)(x-2)(x-2)$, like terms do not add to zero.
$x^3+6x^2+12x+8=(x+2)^3$ because when multiplying $(x+2)^3=(x+2)(x+2)(x+2)$, like terms do not add to zero.

Factor completely.

1. $5t^4 - 80$

GCF 5 :

$= 5\left(\frac{5t^4}{5} - \frac{80}{5}\right)$

$= 5(\underbrace{t^4 - 16}_{\downarrow})$

Difference of two squares

$\sqrt{t^4} = t^2$ $\sqrt{16} = 4$

$= 5\,(\underbrace{t^2 - 4}_{\text{Difference of two squares}})(\underbrace{t^2 + 4}_{\text{Sum of two squares (prime)}})$

$= \boxed{5(t-2)(t+2)(t^2+4)}$

2. $6x^2y^4 - 21x^3y^5 + 3x^2y^6$

GCF $3x^2y^4$

$= 3x^2y^4\left(\frac{6x^2y^4}{3x^2y^4} - \frac{21x^3y^5}{3x^2y^4} + \frac{3x^2y^6}{3x^2y^4}\right)$

$= 3x^2y^4\left(2 - 7xy + y^2\right)$

$= \boxed{3x^2y^4(y^2 - 7xy + 2)}$

notice the one x in the middle? that won't work.

$\underset{-7}{\overset{2}{\times}}$ no #s mult to 2 but add to -7.

③ $x^6 - 64$

difference of two squares

$\sqrt{x^6} = x^3$

$\sqrt{64} = 8$

$$= (\underbrace{x^3 - 8}_{\text{difference of cubes}})(\underbrace{x^3 + 8}_{\text{Sum of cubes}})$$

$\sqrt[3]{x^3} = x \qquad \sqrt[3]{8} = 2$

using formulas: $a \Leftrightarrow x \qquad b \Leftrightarrow 2$

$$= \boxed{(\underbrace{(x-2)(x^2+2x+4)}_{\text{diff}})(\underbrace{(x+2)(x^2-2x+4)}_{\text{sum}})}$$

* Difference of cubes formula
$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

* Sum of cubes formula
$a^3 + b^3 = (\underbrace{a+b}_{\text{linear factor}})(\underbrace{a^2 - ab + b^2}_{\text{quadratic - all degree 2}})$

SIGNS by SOAP

same – opposite – always positive

check by multiplying

$$(x-2)(x^2+2x+4)$$
$$= x^3 + \cancel{2x^2} + \cancel{4x}$$
$$\quad - \cancel{2x^2} - \cancel{4x} - 8$$
$$= x^2 - 8$$

check

$$(x+2)(x^2-2x+4)$$
$$= x^3 - \cancel{2x^2} + \cancel{4x}$$
$$\quad + \cancel{2x^2} - \cancel{4x} + 8$$
$$= x^3 + 8$$

Common mistake: $(x+2)^3 = (x+2)(x+2)(x+2)$

This is NOT the sum of cubes. Check by multiplying

$$(x+2)\underbrace{(x+2)(x+2)}_{\text{FOIL}}$$
$$= (x+2)(x^2+2x+2x+4)$$
$$= (x+2)(x^2+4x+4)$$
$$= x^3 + 4x^2 + 4x$$
$$\quad + 2x^2 + 8x + 8$$
$$= x^3 + 6x^2 + 12x + 8 \quad \neq \quad x^3 + 8 \ !!$$

Ditto $(x-2)^3 = (x-2)(x-2)(x-2)$

This is NOT the difference of cubes. Check by multiplying.

$$(x-2)(x-2)(x-2)$$
$$= (x-2)(x^2-4x+4)$$
$$= x^3 - 4x^2 + 4x$$
$$\quad - 2x^2 + 8x - 8 \quad = x^3 - 6x^2 + 12x - 8 \quad \neq \quad x^3 - 8 \ !!$$

(4) $-25m^2 - 20mn - 4n^2$

GCF -1

$= -(25m^2 + 20mn + 4n^2)$

perfect square trinomial?

$\sqrt{25m^2} = 5m$

$\sqrt{4n^2} = 2n$

check by FOIL

$(5m + 2n)(5m + 2n)$

$= 25m^2 + 10mn + 10mn + 4n^2$

$= 25m^2 + 20mn + 4n^2$ yes!

$= \boxed{-(5m+2n)(5m+2n)}$

$= \boxed{-(5m+2n)^2}$

(5) $x^2y^2 + 7xy + 12$

$\boxed{(xy+3)(xy+4)}$

3 terms = trinomial

leading coefficient 1

$\sqrt{x^2y^2} = xy$

12

3 4

7

factors of 12:

1×12

2×6

3×4 → add to 7

check by FOIL

$x^2y^2 + 4xy + 3xy + 12$ ✓

(6) $2a^2b + 8abx - 6a^2y - 24axy$

GCF $2a$

$= 2a[\underbrace{ab + 4bx}_{\text{GCF 1st two terms}} - \underbrace{3ay - 12xy}_{\text{GCF 2nd two terms using this sign}}]$

4 terms: grouping

$= 2a[b(a+4x) - 3y(a+4x)]$

This $(a+4x)$ is a binomial GCF of remaining two terms — factor it out.

Now the problem has only 2 terms.

$b(a+4x)$ and $-3y(a+4x)$

$= 2a(a+4x)[b \quad -3y \quad]$

$= \boxed{2a(a+4x)(b-3y)}$

Factor by substitution

(7) $2\underset{\sim}{(a+3)}^2 + 5\underset{\sim}{(a+3)} - 7$

squared   not squared

3 terms

1st term: $2(a+3)^2$

2nd term: $5(a+3)$

3rd term: -7

$u = (a+3)$

means

$= 2u^2 + 5u - 7$

3 terms

trinomial

leading coef $\neq 1$

<u>Step 1</u>: Observe recurring stuff which is squared in one term and not squared in another term.
Let $u =$ stuff (unsquared)

$= (2u \quad\quad)(u \quad\quad)$

± 1 ∓ 7

± 7 ∓ 1

<u>Step 2</u>: Factor using u.
Trinomial

<u>Step 2 Method 1</u>: guess and check

using numbers that are chosen to mult to -7
FOIL, check to see if middle term is $5u$.

$(2u+1)(u-7) \rightarrow -14u + u = -13u$ NO

$(2u-1)(u+7) \rightarrow +14u - u = +13u$ No.

$\underline{(2u+7)(u-1)} \rightarrow -2u + 7u = +5u$ YES.

$(2u-7)(u+1) \rightarrow 2u - 7u = -5u$ NO

<u>Step 2 Method 2</u>: "double X" or "a c" method

rewrite middle term + group

a·c

2(−7)

−14

5

seek two numbers that mult to (ac) result but add to b.

$-1, 14 \Rightarrow 13$ no

$\Rightarrow -2, 7 \Rightarrow 5$ yes

$1, -14 \Rightarrow -13$ no

$2, -7 \Rightarrow -5$ no

Rewrite middle term as two like terms using X result

$2u^2 + 5u - 7$

$= \underbrace{2u^2 - 2u} + \underbrace{7u - 7}$

four terms, grouping

$= 2u(u-1) + 7(u-1)$

$= \underline{\underline{(u-1)(2u+7)}}$

$= (a+3-1)(2(a+3)+7)$

$= (a+2)(2a+6+7)$

$= \boxed{(a+2)(2a+13)}$

<u>Step 3</u>: Replace u by original stuff.

simplify if needed

Factor by substitution.

(8) $5x^{-6} + 29x^{-3} - 42$

step 1:

correct $\begin{cases} u = x^{-3} \\ u^2 = (x^{-3})^2 = x^{-6} \end{cases}$ mult ex

incorrect $\begin{cases} u = x^{-6} \\ u^2 = (x^{-6})^2 = x^{-12} \end{cases}$ not in que

$= 5u^2 + 29u - 42$

step 2: Factor result

Option 1: $(5u \quad)(u \quad)$

-1×42	$1 \times (-42)$
-2×21	$2 \times (-21)$
-3×14	$3 \times (-14)$
-6×7	$6 \times (-7)$

combinations to try using Guess and check

Option 2: $5(-42)$
-210
29

want factors of -210 that add to 29.

$-1 \times 210 \to 209$
$-2 \times 105 \to 103$
$-3 \times 70 \to 67$
$-5 \times 42 \to 37$
$-6 \times 35 \to 29$!!
$-7 \times 30 \to 23$
$-10 \times 21 \to 11$
$-14 \times 15 \to +1$

rewrite and group

$\underbrace{5u^2 - 6u} + \underbrace{35u - 42}$

$u(5u-6) + 7(5u-6)$

$(5u-6)(u+7)$

step 3: Replace u by stuff (unsquared)

$= \boxed{(5x^{-3} - 6)(x^{-3} + 7)}$

Factor by substitution

9 $5x^4 + 29x^2 - 42$

$= 5u^2 + 29u - 42$

factors same as 8

$= (5u-6)(u+7)$

$= \boxed{(5x^2-6)(x^2+7)}$

$\cancel{u = x^4}$
$\cancel{u^2 = (x^4)^2 = x^8}$ } wrong

$u = x^2$
$u^2 = (x^2)^2 = x^4$ } correct

neither 6 nor 7 is a perfect square.

① Find the prime factors of 60.

60

6 10

2 3 2 5

$= 2 \cdot 2 \cdot 3 \cdot 5$

$= \boxed{2^2 \cdot 3 \cdot 5}$

Prime numbers are divisible only by 1 and itself
$\Rightarrow$ Smallest numbers we can split another number into are prime numbers.

② Multiply $6x^2(5x+4)$

$= \boxed{30x^3 + 24x^2}$

distribute a monomial

add exponents

③ Factor $30x^3 + 24x^2$

Find the greatest common factor of 30 and 24.

both are divisible by
1
2 } common factors
3
6. } greatest common factor

Find the greatest common factor of x^3 and x^2

both are divisible by x } common factor
x^2 } greatest common factor

$$= 6x^2\left(\frac{30x^3}{6x^2} + \frac{24x^2}{6x^2}\right)$$

factor out

divide inside

$= \boxed{6x^2(5x+4)}$

Why? This tells us lots of factors, though not always prime:

If $x=1$: $30x^3+24x^2 \rightarrow 30(1)^3 + 24(1)^2 = 54$

$6x^2(5x+4) \rightarrow 6\cdot(1)^2(5\cdot1+4) = 6\cdot1\cdot9$ factors of 54

If $x=2$ $30x^3+24x^2 \rightarrow 30(2)^3 + 24(2)^2 = 336$

$6x^2(5x+4) \rightarrow 6\cdot(2)^2(5\cdot2+4) = 6\cdot2\cdot2\cdot14$ factors of 336

If $x=-1$ $30x^3+24x^2 \rightarrow 30(-1)^3 + 24(-1)^2 = -6$

$6x^2(5x+4) \rightarrow 6(-1)^2(5\cdot(-1)+4) = 6\cdot1\cdot(-1)$

④ Multiply $(x+2)(x+5)$

distribute x and distribute 2

$$= x(x+5) + 2(x+5)$$
$$= x^2 + 5x + 2x + 10$$
$$= \boxed{x^2 + 7x + 10}$$

distribute x
distribute 2

OR

F.O.I.L.

⑤ Factor $x^2+7x+10$

want two numbers that add to 7 and multiply to 10.

$$= \boxed{(x+5)(x+2)}$$

multiply to
↓
10
2 5
7
↑
add to

factors of 10
1, 10 → 1+10 = 11 No
2, 5 → 2+5 = 7 YES

⑥ Factor $5x^3 - 30x^2 - 35x$

Greatest Common Factor (GCF) first

$$= 5x\left(\frac{5x^3}{5x} - \frac{30x^2}{5x} - \frac{35x}{5x}\right)$$
$$= 5x(x^2 - 6x - 7)$$
$$= \boxed{5x(x-7)(x+1)}$$

multiply to
↓
−7
−7 +1
−6
↑
add to

1, 7

The signs must be included!

- To multiply to a negative, one factor is (+) and the other (−).
  $(+)(-) = (-)$
  $(-)(+) = (-)$

⑦ Factor $-2xy^3 + 24xy^2 - 54xy$

GCF first − leading term has negative coefficient
factor out negative in GCF ⇒ change all signs.

$$= -2xy\left(\frac{-2xy^3}{-2xy} + \frac{24xy^2}{-2xy} - \frac{54xy}{-2xy}\right)$$
$$= -2xy(y^2 - 12y + 27)$$
$$= \boxed{-2xy(y-3)(y-9)}$$

27
−3 −9
−12

1×27
3×9

SIGNS:
multiply to (+)
add to (−)
⇒ must be $(-)(-) = (+)$
$(-)+(-) = (-)$

⑧ Multiply $(2x-3)(2x-3)=(2x-3)^2$ — distribute $2x$, also distribute -3

$$= 2x(2x-3) - 3(2x-3)$$

$$= 4x^2 - 6x - 6x + 9$$

$$= \boxed{4x^2 - 12x + 9}$$ combine like terms.

when multiplying a binomial by a binomial, we can also use the acronym FOIL = first, outside, inside, last

I L

$$(2x - 3)(2x - 3)$$

F O

$$= 2x \cdot 2x + (2x)(-3) + (-3)(2x) + (-3)(-3)$$

$$= 4x^2 - 6x - 6x + 9$$

$$= \boxed{4x^2 - 12x + 9}$$

⑨ Factor $4x^2 - 12x + 9$

negative

perfect square $2x \cdot 2x$ — perfect square $3 \cdot 3$

$$= (2x - 3)(2x - 3)$$

$$= \boxed{(2x-3)^2}$$

check by F.O.I.L.

$4x^2 - 6x - 6x + 9$

$4x^2 - 6x + 9$ ✓

⑩ Factor $49x^2 + 14x + 1$

positive

perf sq $7x \cdot 7x$ — perf sq $1 \cdot 1$

$$= (7x + 1)(7x + 1)$$

$$= \boxed{(7x+1)^2}$$

check by FOIL

$49x^2 + 7x + 7x + 1$

$49x^2 + 14x + 1$

⑪ Factor $-9x^3 + 30x^2y - 25xy^2$

GCF $\quad -x\left(\frac{-9x^3}{-x} + \frac{30x^2y}{-x} - \frac{25xy^2}{-x}\right)$

$$= -x(9x^2 - 30xy + 25y^2)$$

$$= -x(3x-5y)(3x-5y)$$

$$= \boxed{-x(3x-5y)^2}$$

$9x^2 = 3x \cdot 3x$ perfect square

$25y^2 = 5y \cdot 5y$

check by FOIL

$(3x-5y)(3x-5y)$

$= 9x^2 - 15xy - 15xy + 25y^2$

(12) Multiply $(x+2)(x-2)$

$= x^2 - 2x + 2x - 4$

$= \boxed{x^2 - 4}$

(13) Factor $x^2 - 4$

$= (x+2)(x-2)$

2 terms!
perfect squares!
memorize pattern $a^2 - b^2 = (a-b)(a+b)$ *

(14) Factor $x^2 + 4$

$= \boxed{\text{prime}}$

$(x+2)(x+2)$

$= x^2 + 2x + 2x + 4$

$= x^2 + 4x + 4$ not $x^2 + 4$

(15) Multiply $(x+3)(x^2-4)$

$= x^3 - 4x + 3x^2 - 12$

$= \boxed{x^3 + 3x^2 - 4x - 12}$

(16) Factor $x^3 + 3x^2 - 4x - 12$

4 terms : grouping method

step 1: Factor GCF from first 2 terms only

$x^2(x+3) \quad -4x - 12$

step 2: Factor GCF, using sign on 3rd term of original, from 3rd & 4th

$x^2(x+3) \quad -4(x+3)$

step 3: Check that the ( ) are exactly the same.
If not, start over, rearranging the terms.
-OR- check your arithmetic and signs

Step 4: Factor out GCF

$(x+3)[\,x^2 \quad -4\,]$

$(x+3)(x^2-4)$

check that factors can be factored (x^2-4) is a diff of squares.

$\boxed{(x+3)(x+2)(x-2)}$

Factor completely. (Recall: Factoring is the opposite of multiplying. Check work by multiplying.)

Factoring out a monomial or binomial Greatest Common Factor (GCF) ← largest # or variable with highest exponent that divides all terms.

1) $6xy^4 - 3x^2y^3 + 12x^3y^2$ 3 terms

$= 3xy^2\left(\frac{6xy^4}{3xy^2} - \frac{3x^2y^3}{3xy^2} + \frac{12x^3y^2}{3xy^2}\right)$ ← most people do this division in their heads.

$= \boxed{3xy^2(2y^2 - xy + 4x^2)}$

Terms: separated by add or subtract outside ( )

Note: A fully factored answer has one term: all + or − are inside ( ).

2) $[7x(x^2+5y)+3a(x^2+5y)]$ 2 terms ⇒ each term has common factor (x^2+5y)

$= (x^2+5y)[7x+3a]$

$= \boxed{(x^2+5y)(7x+3a)}$

Factoring by Grouping GCF 1st 2 terms, GCF 2nd 2 terms, GCF in ( ).

3) $x^3+3x^2-4x-12$

$x^2(x+3) - 4(x+3)$

must be the same, polynomial GCF

factor (x+3) from both

$(x+3)(x^2-4)$

$\boxed{(x+3)(x+2)(x-2)}$

x^2-4 is a difference of squares

Factoring trinomials with leading coefficient 1 or with GCF which can be factored out to leave 1

4) $5x^3 - 30x^2 - 35x$

$5x(x^2-6x-7)$ GCF 5x first

$\boxed{5x(x-7)(x+1)}$

−7 ← seek 2 #s that multiply to −7

−7 +1

−6 ← the same 2 #s must add to −6

Factoring trinomials with leading coefficient not equal to 1 and not a GCF

Hint: factorable if b^2-4ac is a (+) perfect square

5) $2x^2+11x+15$

Method 1: Guess and check: rig F&L, check O+I

1×2 1×15, 3×5

$(x+1)(2x+15) \to 17x$ NO

$(x+15)(2x+1) \to 31x$ NO

$(x+3)(2x+5) \to 11x$ YES

$(x+5)(2x+5) \to 15x$ NO

$\boxed{(x+3)(2x+5)}$

Method 2: "double X" to rewrite & group

2·15 ← a·c from ax^2+bx+c

30; 6, 5; 11 ← b

Seek two #s that multiply to 30 and add to 11.

*CAUTION: 6 and 5 are NOT the final answers.

$=2x^2+\underbrace{6x+5x}+15$ rewrite original

must be like terms with original

$=2x(x+3)+5(x+3)$

$=\boxed{(x+3)(2x+5)}$

Factoring trinomials by substitution

temporarily replace a complicated part by u.

6) $2(a+3)^2+5(a+3)-7$ 2(−7), −14; 7, −2; 5

$2u^2+5u-7$ replace (a+3) by u, temporarily

$2u^2+7u-2u-7$

$u(2u+7)-1(2u+7)$

$(2u+7)(u-1)$ return (a+3 in place of u. Hint: change outer ( ) to [ ] can clarify.

$[2(a+3)+7]\,[(a+3)-1]$ dist 2

$[2a+6+7]\,[a+2]$ combine

$\boxed{(2a+13)(a+2)}$

7) $5x^{-6}+29x^{-3}-42$ $u=x^{-3}$ $(u)^2=(x^{-3})^2=x^{-6}$

$5u^2+29u-42$

1×5 1×42, 2×21, 3×14, 6×7

Hint: 42 is large, so guess and check may be quicker.

Hint: List factor pairs in order so you don't skip any.

$(5u-6)(u+7)$

$\boxed{(5x^{-3}-6)(x^{-3}+7)}$

Perfect square trinomials Memorize: $a^2+2ab+b^2=(a+b)^2$ $a^2-2ab+b^2=(a-b)^2$

8) $m^2 \underset{2\cdot(-5)}{-10m} + \underset{5^2}{25}$

$\boxed{(m-5)(m-5)}$ or $\boxed{(m-5)^2}$

Difference or sum of squares Memorize: $a^2-b^2=(a-b)(a+b)$ a^2+b^2 is prime.

9) $32-162t^4$

$= 2(16-81t^4)$ GCF

$= 2(4-9t^2)(4+9t^2)$ difference of squares $16=4^2$ and $81t^4=(9t^2)^2$

$= \boxed{2(2-3t)(2+3t)(4+9t^2)}$ difference of squares $(4-9t^2)$
Sum of squares $(4+9t^2)$ is prime.

10) $32+162t^4$

$= \boxed{2(16+81t^4)}$ GCF

sum of squares is prime

Note: $16+81t^4$ is prime, but $32+162t^4$ is not prime because it has a GCF

11) $(x+3)^2-36$ Difference of squares

$= [(x+3)-6][(x+3)+6]$

$= [x-3][x+9]$

$= \boxed{(x-3)(x+9)}$

$a^2-b^2=(a-b)(a+b)$
Here $a=x+3$
$b=6=\sqrt{36}$

12) $-49z^6+81z^4$

Method 1: Factor out -1. w/ GCF

$= -z^4(49z^2-81)$

$= \boxed{-z^4(7z-9)(7z+9)}$ diff of sq.

Method 2: change order

$81z^4-49z^6$

$= z^4(81-49z^2)$ GCF

$= \boxed{z^4(9-7z)(9+7z)}$ diff of sq.

Difference or sum of cubes Memorize: method or formula

$a^3-b^3=(a-b)(a^2+ab+b^2)$ $a^3+b^3=(a+b)(a^2-ab+b^2)$

Signs: SOAP

13) y^3-64 cube roots $\sqrt[3]{y^3}=y$ and $\sqrt[3]{64}=4$ $\Rightarrow$ $a=y$ $b=4$ in formula

$= \boxed{(y-4)(y^2+4y+16)}$

S — same sign as original

O — opposite sign from original

AP — always positive whether it's a difference or sum of cubes.

Note the trinomial factor a^2+ab+b^2 or a^2-ab+b^2 is always prime.

14) $375q^2+3n^3q^2$

$= 3q^2(125+n^3)$ GCF

$= 3q^2(n^3+125)$ addition is commutative

$= \boxed{3q^2(n+5)(n^2-5n+25)}$ $\sqrt[3]{n^3}=n$ (a) $\sqrt[3]{125}=5$ (b)

Factoring out a monomial or binomial Greatest Common Factor (GCF)

15) $x^{2/7}-3x^{1/7}$ lowest exponent $\frac{1}{7}<\frac{2}{7}$ $\Rightarrow$ GCF $= x^{1/7}$

$= x^{1/7}\left(\dfrac{x^{2/7}}{x^{1/7}} - 3\dfrac{x^{1/7}}{x^{1/7}}\right)$ subtract exponents $\frac{2}{7}-\frac{1}{7}=\frac{1}{7}$

$= \boxed{x^{1/7}(x^{1/7}-3)}$

---

16) $3x^{-3/5}-3x^{2/5}$ lowest exp $-3/5 < 2/5$ means GCF $= x^{-3/5}$ (with coefficient 3)

$= 3x^{-3/5}\left[\dfrac{3x^{-3/5}}{3x^{-3/5}} - \dfrac{3x^{2/5}}{3x^{-3/5}}\right]$ subtract exponents: $2/5-{-3/5} = 2/5+3/5 = 5/5 = 1$

$x^1 = x$

$= \boxed{3x^{-3/5}[1-x]}$